

Application of Two-Variable Fuzzy-PI Control in an Aeroengine

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This paper proposes a two-variable fuzzy control algorithm, synthesizes a new two-variable fuzzy-PI controller, and studies the decoupling characteristic and responsibility of a controller in an aeroengine by computer simulation. The results show that the fuzzy-PI controller has a static decoupling characteristic and satisfied responsibility. The rules of a two-variable control system in aeroengines are also probed, and a new method for engine control is provided.

Introduction

THE aeroengine is a high-order, strong, nonlinear, time-varying complex system. The property and parameter of aeroengines vary widely and indefinitely when working conditions change; it is therefore difficult to set up a highly precise mathematical model and to design a high-quality controller. In other words, there must be more effort to obtain a high-quality controller.

Because fuzzy control possesses strong robustness, and a mathematics model of the control plant or a precise mathematics model of it is not necessary, a fuzzy control algorithm is convenient and flexible. Fuzzy control, it is hoped, will overcome the indefinite factor of setting a control model of an aeroengine and will get better control performance.

The application of fuzzy control theory in the rotational speed control of an aeroengine has been studied,^{1–3} and an experimental investigation has also been done on an aeroengine.⁴ This research has only been done on a single-input single-output system. Taking an aeroengine as a control plant, this paper studies the properties of a fuzzy controller and the methods of multivariables of fuzzy control based on fuzzy reasoning. To meet the need of a strong coupling aeroengine, a new two-variable fuzzy control algorithm is proposed in this paper, a new two-variable fuzzy-PI controller is synthesized, and the application of the controller in an aeroengine is studied. The rules of a two-variable control system in an aeroengine are also probed and a new method of engine control is provided.

Study of Two-Variable Fuzzy Control Based on Fuzzy Reasoning

The study of multivariable control law is important in the field of aeroengine control. Many specialists have done a great deal of work and have concluded the basic rules. This paper adopted a rule that the rotation speed of a low-pressure rotor N_l is controlled at a constant value by fuel flow M_f , and the gas temperature behind the turbine T_4^* is regulated at a constant value by nozzle area A_e . That is,

$$M_f \rightarrow N_l = \text{const}$$

$$A_e \rightarrow T_4^* = \text{const}$$

This control law is not two single-input, single-output loops, but is a complex strong coupling system in which the two

variables are interactive and affect each other. The feasibility of adopting multivariable fuzzy control in aeroengines is studied.

Design Rule of Fuzzy Control

Fuzzy variables EN_L , ET_4^* , ΔM_f , and ΔA_e are divided into seven language values: NB, NM, NS, Z, PS, PM, and PB, and the universe of discourse is classified to 15 degrees: $-7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7$. The membership function of every fuzzy variable is listed in Table 1.

According to the principle of an aeroengine and its control law, we conclude 49 control rules. They are

$$\text{If } EN_{L(1)} = \text{NB and } ET_{4(1)}^* = \text{PB}$$

$$\text{then } \Delta M_{f(1)} = \text{NB and } \Delta A_{e(1)} = \text{PB}$$

...

$$\text{If } EN_{L(i)} = \text{PS and } ET_{4(i)}^* = \text{NM}$$

$$\text{then } \Delta M_{f(i)} = \text{NM and } \Delta A_{e(i)} = \text{PS}$$

...

$$\text{If } EN_{L(49)} = \text{PB and } ET_{4(49)}^* = \text{PB}$$

$$\text{then } \Delta M_{f(49)} = \text{PB and } \Delta A_{e(49)} = \text{NM}$$

Design of Control Algorithm

A two-variable fuzzy-PI control algorithm is made up of two parts: 1) fuzzy reasoning algorithm and 2) PI algorithm. The fuzzy relationship matrix of a fuzzy control system is shown as the following equation:

$$R = \bigcup_{i=1}^{49} \{EN_{L(i)} ET_{4(i)}^* \Delta M_{f(i)} \Delta A_{e(i)}\}$$

R is a $q_1 \times q_2 \times q_3 \times q_4 = 15^4$ -dimension fuzzy relationship matrix; the fuzzy relationship equation is

$$Y = EN_L \cdot ET_4^* \cdot R$$

Here, $Y = [\Delta m_f \Delta A_e]^T$. By a multidimensional fuzzy decomposition method⁵ we decomposed the fuzzy relationship matrix R to four small dimensions, R_{11} , R_{21} , R_{12} , and R_{22} , and defined the fuzzy relationship equation of the fuzzy controller as follows:

$$\begin{aligned} \Delta M_f &= EN_L \cdot R_{11} \cap ET_4^* \cdot R_{21} \\ \Delta A_e &= EN_L \cdot R_{12} \cap ET_4^* \cdot R_{22} \end{aligned} \quad (1)$$

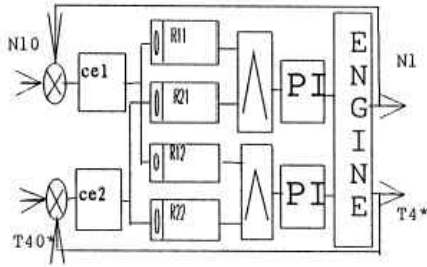
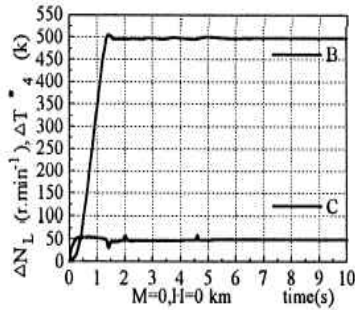
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Table 1 Membership function table of fuzzy variables

Language value	Universe of discourse														
	-7	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6	7
NB	1.0	0.8	0.5	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NM	0.5	0.8	1.0	0.8	0.5	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
NS	0.0	0.1	0.3	0.5	0.8	1.0	0.8	0.5	0.3	0.1	0.0	0.0	0.0	0.0	0.0
Z	0.0	0.0	0.0	0.1	0.3	0.5	0.8	1.0	0.8	0.5	0.3	0.1	0.0	0.0	0.0
PS	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.5	0.8	1.0	0.8	0.5	0.3	0.1	0.0
PM	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.5	0.8	1.0	0.8	0.5
PB	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.3	0.5	0.8	1.0

**Fig. 1** Structure of two-variable fuzzy-PI control system based on fuzzy reasoning.**Fig. 2** Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$ and $\Delta T_4^* = 50 \text{ K}$.

The small-dimension fuzzy relationship matrices R_{11} , R_{21} , R_{12} , and R_{22} are calculated off line; the algorithms of R_{11} , R_{21} , R_{12} , and R_{22} are as follows:

$$R_{11} = \bigcup_{i=1}^{49} \{EN_{L(i)} \cap \Delta M_{f(i)}\}$$

$$R_{12} = \bigcup_{i=1}^{49} \{EN_{L(i)} \cap \Delta A_{c(i)}\}$$

$$R_{21} = \bigcup_{i=1}^{49} \{ET_{4(i)}^* \cap \Delta M_{f(i)}\}$$

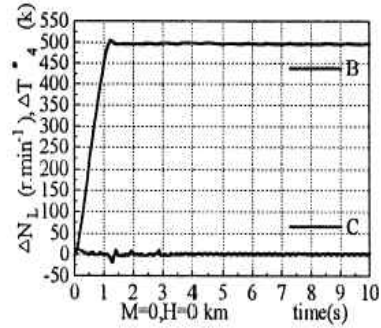
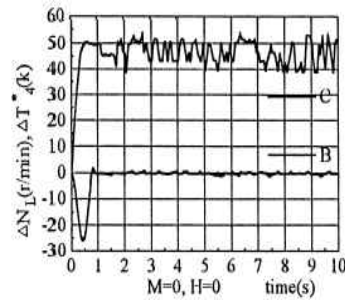
$$R_{22} = \bigcup_{i=1}^{49} \{ET_{4(i)}^* \cap \Delta A_{c(i)}\}$$

The fuzzy relationship matrices R_{11} , R_{21} , R_{12} , and R_{22} calculated by the preceding equations are 15×15 -dimension; the values of their elements are real data, and are less than 1, such as R_{11} , and the fuzzy control variables ΔM_f and ΔA_c are calculated by Eq. (1) on line.

The defuzzification of ΔM_f and ΔA_c adopts a weighted average and takes the membership function of a control variable as a weighted function; the algorithm is

$$\Delta M = \frac{\sum_{j=1}^{15} \Delta M_f(j) \cdot U_{\Delta M_f(j)}}{\sum_{j=1}^{15} U_{\Delta M_f(j)}}$$

$$\Delta A = \frac{\sum_{j=1}^{15} \Delta A_c(j) \cdot U_{\Delta A_c(j)}}{\sum_{j=1}^{15} U_{\Delta A_c(j)}}$$

**Fig. 3** Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$ and $\Delta T_4^* = 0 \text{ K}$.**Fig. 4** Inputs: $\Delta N_L = 0 \text{ rpm}^{-1}$ and $\Delta T_4^* = 50 \text{ K}$.

$$R_{11} = \begin{bmatrix} 1.0 & .80 & .50 & .50 & .50 & .30 & .10 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .80 & .80 & .80 & .80 & .50 & .30 & .10 & .10 & .10 & .00 & .00 & .00 & .00 & .00 & .00 & .00 \\ .50 & .80 & 1.00 & .80 & .50 & .30 & .30 & .30 & .30 & .10 & .00 & .00 & .00 & .00 & .00 & .00 \\ .50 & .80 & .80 & .80 & .50 & .50 & .50 & .50 & .30 & .10 & .10 & .10 & .00 & .00 & .00 & .00 \\ .50 & .50 & .50 & .50 & .80 & .80 & .80 & .50 & .30 & .30 & .30 & .10 & .00 & .00 & .00 & .00 \\ .30 & .30 & .30 & .50 & .80 & 1.0 & .80 & .50 & .50 & .50 & .30 & .10 & .10 & .10 & .00 & .00 \\ .10 & .10 & .30 & .50 & .80 & .80 & .80 & .80 & .50 & .30 & .30 & .30 & .10 & .10 & .00 & .00 \\ .00 & .10 & .30 & .50 & .50 & .50 & .80 & 1.0 & .80 & .50 & .50 & .30 & .10 & .10 & .00 & .00 \\ .00 & .10 & .30 & .30 & .30 & .50 & .80 & .80 & .80 & .80 & .50 & .30 & .10 & .10 & .00 & .00 \\ .00 & .10 & .10 & .10 & .30 & .50 & .50 & .50 & .80 & 1.0 & .80 & .50 & .30 & .30 & .30 & .30 \\ .00 & .00 & .00 & .10 & .30 & .30 & .30 & .50 & .80 & .80 & .80 & .50 & .50 & .50 & .50 & .50 \\ .00 & .00 & .00 & .10 & .10 & .10 & .30 & .50 & .80 & .80 & .80 & .80 & .80 & .80 & .80 & .50 \\ .00 & .00 & .00 & .00 & .00 & .10 & .30 & .50 & .80 & 1.0 & .80 & .80 & .80 & .80 & .80 & .80 \\ .00 & .00 & .00 & .00 & .00 & .10 & .30 & .50 & .50 & .50 & .50 & .80 & 1.0 & .80 & 1.00 & .00 \end{bmatrix}$$

Here, $\Delta M_f(j)$ and $\Delta A_c(j)$ are the elements of a universe of discourse, and $U_{\Delta M_f(j)}$ and $U_{\Delta A_c(j)}$ are the membership functions

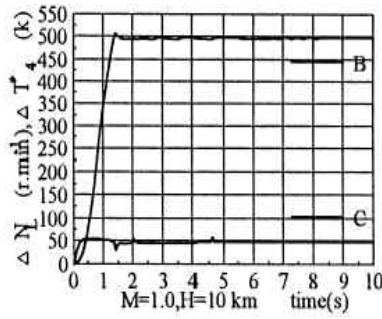


Fig. 5 Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$ and $\Delta T_4^* = 50 \text{ K}$.

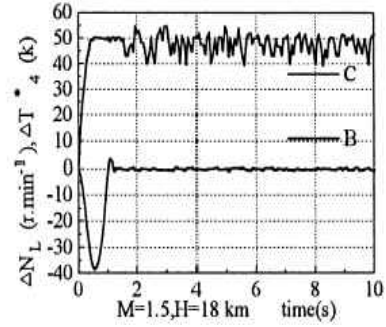


Fig. 8 Inputs: $\Delta N_L = 0 \text{ rpm}^{-1}$, $\Delta T_4^* = 50 \text{ K}$.

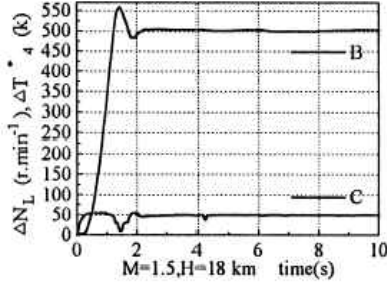


Fig. 6 Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$ and $\Delta T_4^* = 50 \text{ K}$.

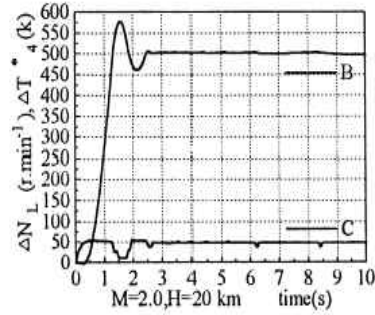


Fig. 9 Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$ and $\Delta T_4^* = 50$.

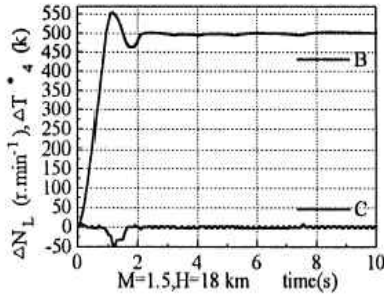


Fig. 7 Inputs: $\Delta N_L = 500 \text{ rpm}^{-1}$, $\Delta T_4^* = 0 \text{ K}$.

corresponding to the element. The PI part of the algorithm is the following:

$$M_f = c_{11} \int_0^r \Delta M \times dt + c_{12} \times \Delta M$$

$$A_e = c_{21} \int_0^r \Delta A \times dt + c_{22} \times \Delta A$$

The M_f and A_e obtained from the calculation are the control values of outputs of the fuzzy-PI controller. The coefficients C_{11} , C_{12} , C_{21} , and C_{22} of the algorithm come from the testing ground.

Design and Simulation of Circuit Loop Fuzzy Control System

By the parameters, the fuzzy-PI algorithm that is determined and the control law selected earlier, the control system designed is shown as Fig. 1. Figure 1 is the structure of a two-variable controller based on fuzzy reasoning with two inputs; the control algorithm is a two-variable fuzzy-PI method; the errors EN_i and ET_4^* of the increment of controlled variables, the rotational speed of low-pressure rotor ΔN_L , and the temperature behind turbine ΔT_4^* are inputs of the controller; and

the increments of control variables, fuel flow M_f , and nozzle area A_e are outputs of the fuzzy controller.

A small divagation linear model (increment form) of an aeroengine is set up by a differential method; the state-space model is

$$\begin{bmatrix} \dot{n}_H \\ \dot{n}_L \end{bmatrix} = A \begin{bmatrix} \Delta n_H \\ \Delta n_L \end{bmatrix} + B \begin{bmatrix} \Delta M_f \\ \Delta A_c \end{bmatrix} \quad (3)$$

$$\begin{bmatrix} \Delta n_L \\ \Delta T_4^* \end{bmatrix} = C \begin{bmatrix} \Delta n_H \\ \Delta n_L \end{bmatrix} + D \begin{bmatrix} \Delta M_f \\ \Delta A_c \end{bmatrix}$$

A , B , C , and D are coefficient matrices.

Using the preceding model equations, the simulation of the system in Fig. 1 has been done at many conditions among the full-flight envelope: the simulated curves of several conditions are shown in Figs. 2–9; curve B is the rotor speed response and curve C is the temperature. Figures 2, 5, 6, and 9 show that the fuzzy control system is able to track the input signals well and obtain the control precision needed, though the parameters of the model vary greatly among the entire flight envelope. Figures 3, 4, 8, and 9 show that the system has a good static decouple characteristic and responsibility, and the controller's easy implementation compares to a conventional gain-scheduled PI controller in this complex two-input, two-output system. This fuzzy-PI controller does not need a special uncoupling mechanism; the control algorithm complexity was reduced and this compared to the conventional gain-scheduled PI controller, where the system is more simple.

Conclusions

The simulated results show that two-variable fuzzy-PI control based on fuzzy reasoning has good static decoupling characteristics and quick responsibility, its structure is reasonable and feasible, and it is a better fuzzy controller. An aeroengine fuzzy-PI control system can meet the needs of control precision and quick responsibility.

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